HL Measurement Practice Questions
03/01/2007

Student Name: ______________________________

Class: ______________________________

Date: ______________________________

Instructions: **Read each question carefully and select the correct answer.**

1. Find the volume of a sphere with diameter = 7 inches. Round your answer to the nearest cubic inch. Use π = 3.14.

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

A. 1,436 in^3
B. 180 in^3
C. 3,146 in^3
D. 360 in^3

2. Calculate the volume of a sphere with radius of 6.7 feet. Round to the nearest cubic foot. Use π = 3.14.

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

A. 10,074 ft^3
B. 157 ft^3
C. 1,259 ft^3
D. 277 ft^3

3. Determine the volume of a sphere with radius 8.8 m. Round to the nearest cubic meter. Use π = 3.14.

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

A. 2,853 m^3
B. 6,251 m^3
C. 357 m^3
D. 50,010 m^3

4. Determine the volume of a sphere with radius = 3.7 m. Round your answer to the nearest cubic meter. Use π = 3.14.

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

A. 27 m^3
B. 212 m^3
C. 1,697 m^3
D. 3,717 m^3

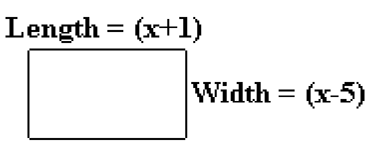
5. A rectangle has a width that is 5 more inches than its length. The area of the rectangle is 150 square inches. What is the width of the rectangle?

A. 15 inches
B. 10 inches
C. 20 inches
D. 25 inches

6. The area of a rectangle is 56 square feet. The length is (x - 2) feet and the width is (x + 8) feet. What is the length of the rectangle?

A. 8 feet
B. 14 feet
C. 4 feet
D. 10 feet

7. The area of the following rectangle is 55 square feet. What is the width of the rectangle?



A. 11 feet
B. 1 foot
C. 7 feet
D. 5 feet

8. What is the maximum value given the following constraints?

Objective Quantity:

$C = 6x + 3y$

Constraints:

$x \geq 0$

$y \geq 0$

$2x + 8y \geq 8$

$2x + 2y \geq 4$

A. 6
B. 10
C. 12
D. No maximum

9. Calculate the volume of a cone with a height of 12.2 cm and a base that is 9 cm in diameter. (Use pi = 3.14)

Volume Formulas	
$V = \frac{1}{3} \cdot B \cdot h$	$V = \frac{4}{3} \cdot \pi \cdot r^3$
$V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$	$V = \pi \cdot r^2 \cdot h$
$V = B \cdot h$	

A. 1,034 cm^3
B. 115 cm^3
C. 776 cm^3
D. 259 cm^3

10. Find the major axes of an ellipse whose minor axes is 8 inches and whose area is 20 square inches.

Round your answer to the nearest hundredth.

A. 24.66/pi inches
B. 4.96/pi inches
C. 5/pi inches
D. 10/pi inches

11. Find the area of a circle with a diameter of 12 m. Express your answer in terms of π.

A. 6π m^2

B. 12π m^2

C. 144π m^2

D. 36π m^2

12. Find the diameter of a circle whose circumference is 95 feet.

A. 95/pi feet
B. 190/pi feet
C. 47.5/pi feet
D. 44.99/pi feet

HL Measurement Practice
Answer Key
03/01/2007

1. Find the volume of a sphere with diameter = 7 inches. Round your answer to the nearest cubic inch. Use π = 3.14.

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

B. 180 in^3
Volume of Spheres

2. Calculate the volume of a sphere with radius of 6.7 feet. Round to the nearest cubic foot. Use π = 3.14.

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

C. 1,259 ft^3
Volume of Spheres

3. Determine the volume of a sphere with radius 8.8 m. Round to the nearest cubic meter. Use π = 3.14.

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

A. 2,853 m^3
Volume of Spheres

4. Determine the volume of a sphere with radius = 3.7 m. Round your answer to the nearest cubic meter. Use π = 3.14.

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

B. 212 m^3
Volume of Spheres

5. A rectangle has a width that is 5 more inches than its length. The area of the rectangle is 150 square inches. What is the width of the rectangle?

A. 15 inches
Evaluating Solutions - B

6. The area of a rectangle is 56 square feet. The length is (x - 2) feet and the width is (x + 8) feet. What is the length of the rectangle?

C. 4 feet
Evaluating Solutions - B

7. The area of the following rectangle is 55 square feet. What is the width of the rectangle?

Length = (x+1)

Width = (x-5)

D. 5 feet
Evaluating Solutions - B

8. What is the maximum value given the following constraints?

Objective Quantity:

$C = 6x + 3y$

Constraints:

$x \geq 0$

$y \geq 0$

$2x + 8y \geq 8$

$2x + 2y \geq 4$

D. No maximum
Evaluating Solutions - B

9. Calculate the volume of a cone with a height of 12.2 cm and a base that is 9 cm in diameter. (Use pi = 3.14)

Volume Formulas	
$V = \frac{1}{3} \cdot B \cdot h$	$V = \frac{4}{3} \cdot \pi \cdot r^3$
$V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$	$V = \pi \cdot r^2 \cdot h$
$V = B \cdot h$	

D. 259 cm^3
Irrational Numbers: Pi

10. Find the major axes of an ellipse whose minor axes is 8 inches and whose area is 20 square inches.

Round your answer to the nearest hundredth.

D. 10/pi inches
Irrational Numbers: Pi

11. Find the area of a circle with a diameter of 12 m. Express your answer in terms of π.

D. 36π m^2
Irrational Numbers: Pi

12. Find the diameter of a circle whose circumference is 95 feet.

A. 95/pi feet
Irrational Numbers: Pi

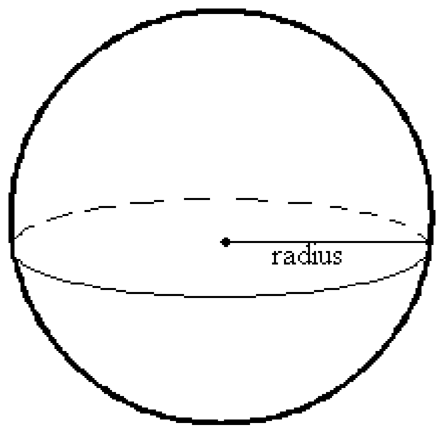
Study Guide

HL Measurement Practice
03/01/2007

Volume of Spheres
This study guide will focus on finding the volume of a sphere.

Remember the volume is the measure of the amount of space inside an object.

A sphere is a three-dimensional circle.



Sphere

When a student is asked to find the volume of a sphere, he or she will be given the radius, the distance between the center of a sphere and any point on the sphere , or the diameter, the distance between any two points on the sphere passing through the center. They will also need the following formula which may or may not be given:

$$V = \frac{4}{3}\pi r^3 \quad \text{where } \pi = 3.14$$

$$r = \text{radius}$$

To solve, substitute the radius (divide the diameter by two to get the radius) for *r* and 3.14 for π. Then use order of operations to find the answer. A quick review of order of operations is provided below.

P E M D A S

P = Parentheses, E = Exponents, M = Multiplication, D = Division, A = Addition, S = Subtraction.
**Note: Evaluate multiplication and division from left to right (whichever comes first). Evaluate addition and subtraction from left to right (whichever comes first).

Example 1: Calculate the volume of a sphere with radius 9.5 m. Round to the nearest cubic meter. Use π = 3.14.

(1) $V = \frac{4}{3}(3.14)(9.5)^3$ (2) $V = \frac{4}{3}(3.14)(857.375)$ (3) $V = 3{,}589.54\overline{3}\ \text{m}^3$

(4) $V = 3{,}590\ \text{m}^3$

Step 1: Substitute 9.5 m in for the radius and 3.14 for π.
Step 2: Evaluate the exponent first by taking 9.5 to the third power to get 857.375.
Step 3: Multiply across to get 3,589.543...
Step 4: Round to the nearest cubic meter to get 3,590 m^3.

Answer: 3,590 m^3

Example 2: Find the volume of a baseball that is 2.8 inches in diameter. Round your answer to the nearest cubic inch.

(1) $\frac{2.8}{2} = 1.4$ (2) $V = \frac{4}{3}(3.14)(1.4)^3$ (3) $V = \frac{4}{3}(3.14)(2.744)$

(4) $V = 11.48821\overline{3}\ \text{in}^3$ (5) $V = 11\ \text{in}^3$

Step 1: Find the radius. Remember that the radius is half of the diameter so 2.8 ÷ 2 = 1.4 inches.
Step 2: Substitute 1.4 for *r* and 3.14 for π.
Step 3: Evaluate the exponent first by taking 1.4 to the third power to get 2.744.
Step 4: Multiply across to get 11.4882333...
Step 5: Round to the nearest cubic inch to get 11 in^3.

Answer: 11 in^3

An activity that would help reinforce this skill would be to have the student research the different diameters or radii of spherical objects like sporting equipment or the planets. Have him or her estimate the volume of the different objects. Then have the student verify his or her prediction by calculating the actual volume using the formula.

Evaluating Solutions - B

The Evaluating Solutions skill asks students to determine the reasonableness of possible problem solutions.

Determining the area of a rectangle and a square:

The area of a rectangle is given by the following formula:

$$A = l \cdot w$$

where l represents the length of the rectangle and w represents the width of the rectangle. The unit of measure for area is the square of the units given for the length and width. For example, if the length of a rectangle is 8 inches and the width is 6 inches, then the area would be

$$A = l \cdot w = 8 \cdot 6 = 48$$

The area of the given rectangle is 48 square inches.

A square is a special type of rectangle in which the length and the width are congruent. Thus, the area of a square is given by the following formula:

$$A = s^2$$

where "s" is the length of a side of the square. The unit of measure for area is the square of the units given for the length of a side. If the length of a square is 5 feet, then the area of the square would be

$$A = s^2 = 5^2 = 5 \cdot 5 = 25$$

The area of the given square is 25 square feet.

Determining the length of a side of a square, given its area:

When finding the length of a side of a square, given its area, find the square root of a number, it may be necessary to use the calculator to so. The square root key is indicated by the symbol $\sqrt{\ }$. On some calculators it is necessary to enter the number first and then the square root key to obtain the answer. On other calculators you may need to enter the square root key first, followed by the number key. It may be necessary to press SHIFT, 2nd, or INV before the $\sqrt{\ }$.

Example 1: The area of a square is 81 square feet. The length of a side is x inches. What is the length of each side?

(1)	(2)	(3)	(4)
$A = s^2$	$81 = s^2$	$\pm\sqrt{81} = \sqrt{s^2}$ $\pm 9 = s$	**The length of each side is 9 inches.**

Step 1: Write down the correct area formula.
Step 2: Substitute 81 in place of A.
Step 3: Take the square root of both sides. The square roots of 81 are +9 and - 9.
Step 4: Only the positive square root can be used when solving the word problem. (Negative numbers can not be used to represent the length of sides of a geometric figure.)

Determining the length or width of a rectangle, given its area:

Example 2: The area of a rectangle is 24 square inches. The length is (x + 3) inches and the width is (x - 2) inches. What is the length of the rectangle?

(1) $A = l \cdot w$

(2) $24 = (x+3)(x-2)$

(3) $24 = x^2 + x - 6$

(4)
$$\begin{array}{r} 24 = x^2 + x - 6 \\ -24 \qquad\qquad -24 \\ \hline 0 = x^2 + x - 30 \end{array}$$

(5) $0 = (x+6)(x-5)$

(6)
$$\begin{array}{r} (x+6) = 0 \\ -6 \quad -6 \\ \hline x = -6 \end{array} \quad \text{or} \quad \begin{array}{r} (x-5) = 0 \\ +5 \quad +5 \\ \hline x = 5 \end{array}$$

(7) **The length of the rectangle is** $x + 3 = 5 + 3 = 8$ **inches.**

Step 1: Write down the correct area formula.
Step 2: Substitute the given values and expressions into the formula. A = 24, l = (x + 3) and w = (x - 2)
Step 3: Multiply the binomials using the distributive property or the **FOIL** method and simplify. (Recall that in the **FOIL** method, multiply the **F**irst elements in each factor of the binomial to get (x)(x) =

x^2. Next multiply the pairs of **O**uter and **I**nner elements to get (3)(x) = 3x and (x)(-2) = -2x, respectively. Combine the like terms 3x + (-2x) to get x. Then multiply the **L**ast elements to get (3)(-2) = -6.)
Step 4: Add -24 to both sides of the equation to get 0 on one side of the equal sign.
Step 5: Factor the trinomial. Recall that to factor a trinomial, undo the process of the FOIL method.
• The "x" in each binomial factor came from $x^2 = (x)(x)$. • To get the other elements of the binomial factors, look for two numbers which when multiplied will give -30, but when added will equal 1, the coefficient of the x term in the trinomial $x^2 + x - 30$. The two numbers that satisfy those two conditions are 6 and -5.
Step 6: Set each of the factors equal to zero, so x + 6 = 0 or x - 5 = 0. Solve to get the two answers, -6 and 5.
Step 7: Only the positive value for x can be used when solving the word problem. The expression (x + 3) represents the length of the rectangle, thus substitute 5 in place of the x and simplify.

Answer: 8 inches

If the above example had asked for the width of the rectangle, then Step 7 would have been written as follows:

(7) The width of the rectangle is (x - 2) = 5 - 2 = 3 inches.

Step 7: Only the positive value for x can be used when solving the word problem. The expression (x - 2) represents the length of the rectangle, thus substitute x by 5 and simplify.

Example 3: A rectangle has a width that is 3 more inches than its length. The area of the rectangle is 70 square inches. What is the width of the rectangle?

(1)
Let x = length of rectangle
Let x+3 = width of rectangle

(2) $A = l \cdot w$

(3) $70 = (x)(x + 3)$

(4) $70 = x^2 + 3x$

(5)
$$\begin{array}{r} 70 = x^2 + 3x \\ -70 \qquad\quad -70 \\ \hline 0 = x^2 + 3x - 70 \end{array}$$

(6) $0 = (x + 10)(x - 7)$

(7)
$$\begin{array}{l} x + 10 = 0 \text{ or } x - 7 = 0 \\ \;\;\, -10 \;\; -10 \qquad\quad +7 \;\; +7 \\ \hline x = -10 \text{ or } x = 7 \end{array}$$

(8)
The width of the rectangle i
$x + 3 = 7 + 3 = 10$ **inches.**

Step 1: Set up the correct "Let" statement to identify the unknown values in the problem. It is easier to let x represent the smaller amount (length) and since the width is 3 more inches than the length, the width is represented by x + 3.
Step 2: Write down the correct area formula.
Step 3: Substitute the given values and expressions into the formula. A = 70, l = (x) and w = (x + 3).
Step 4: Multiply using the distributive property. Multiply $(x)(x) = x^2$ and $(x)(3) = 3x$ Step 5: Add -70 to both sides of the equation to get 0 on one side of the equal sign.
Step 6: Factor the trinomial. (Refer to Step 5 in he previous example, if needed.)
Step 7: Set each of the factors equal to 0 and solve to get the two answers, -10 and 7.
Step 8: Only the positive value for x can be used when solving the word problem. The expression (x + 3) represents the width of the rectangle, thus substitute 7 in place of x and simplify.

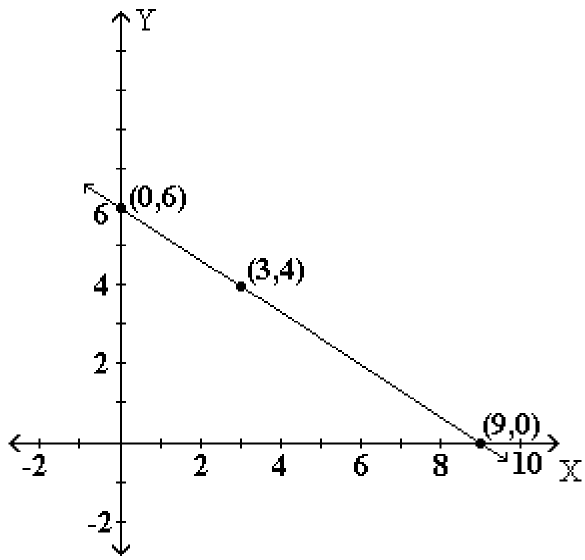
Maximizing or minimizing a function given specific constraints:

The process of maximizing or minimizing a given function, sometimes called an objective quantity, is subject to specific constraints when used in different application problems. The constraints are often expressed as linear inequalities and one method used to solve this type of problem is called linear programming.

To solve this type of problem, it is necessary to graph the given constraints or inequalities. Two constraints that are commonly given are $x \geq 0$ and $y \geq 0$. These constraints limit the graphs of the remaining inequalities to lie only in the first quadrant (where the x and y values are positive as well as the positive x-axis and y-axis values).

Next, graph the other linear inequalities. Recall that to graph an inequality, first sketch a graph of the corresponding equality to get the boundary line for the resulting shaded region. For example, first sketch the corresponding equality 2x + 3y = 18 to graph $2x + 3y \geq 18$ Recall that to sketch 2x + 3y = 18, it is helpful to use an x/y chart (table of values) to find three ordered pairs that lie on the graph of the linear equation. Then plot the three points and draw the line that contains them to get the graph of 2x + 3y = 18.

x	y
0	6
9	0
3	4



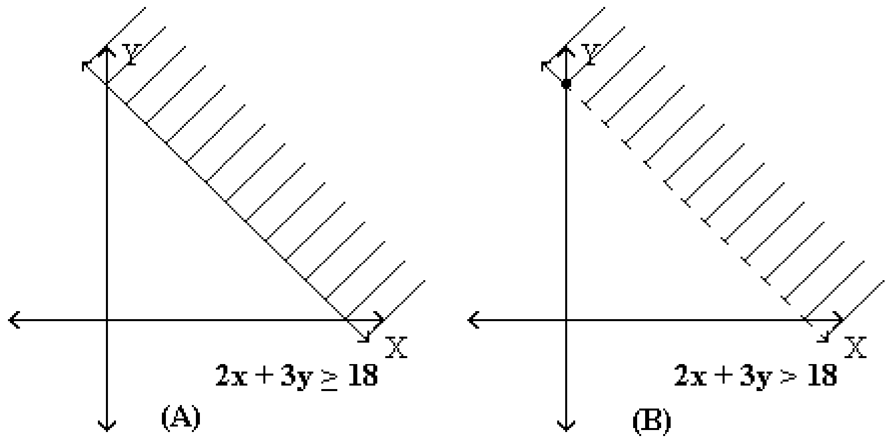
This line divides the rectangular coordinate system into two half-planes. The graph of the solution to an inequality is a shaded region, or half-plane, either above or below the line. To determine which region is correct, pick a test point that does not lie on the graphed line, (often (0, 0) is used), and substitute this point into the inequality to determine if it makes the inequality a true or false statement.

$$2x + 3y \geq 18$$

$$2(0) + 3(0) \geq 18 \qquad \text{Substitute } x = 0 \text{ and } y = 0$$

$$0 \geq 18 \qquad \text{False statement}$$

The point (0, 0) does not lie in the half-plane that represents the solution set to the linear inequality because 0 is NOT greater than 18. Thus the other half-plane is shaded to represent the correct graph to the inequality as shown below by graph A.



The graph of the corresponding equality would have been a dotted line (not a solid one) if the given linear inequality had been $2x + 3y > 18$ see graph B above.

Example 4: What is the minimum value given the following constraints?

Objective Quantity:

$C = 4x + 5y$

Constraints:

$x \geq 0$

$y \geq 0$

$2x + 3y \geq 18$

$2x + y \geq 10$

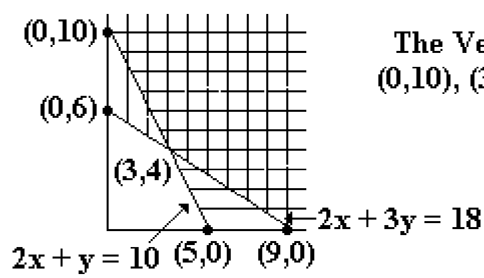
(1)

$$\begin{array}{r} 2x + 3y = 18 \\ 2x + y = 10 \\ \hline 2y = 8 \\ y = 4 \end{array}$$

(2)

$$\begin{array}{r} 2x + 3y = 18 \\ 2x + 3(4) = 18 \\ 2x + 12 = 18 \\ \hline 2x = 6 \\ x = 3 \end{array}$$

(3)



(4)

The Vertices are:
(0,10), (3,4), and (9,0)

(5)

$C = 4x + 5y$

(0,10) $C = 4(0) + 5(10) = 50$

(3,4) $C = 4(3) + 5(4) = 52$

(9,0) $C = 4(9) + 5(0) = 36$

(6)

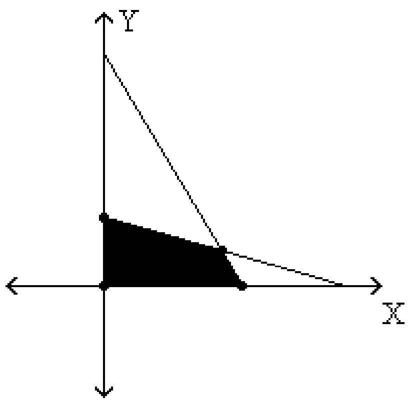
The minimum Value is 36.

<u>Step 1</u>: Solve the system of corresponding equalities to find the point of intersection of the boundary lines. First subtract the bottom equation from the top one (or you could multiply the bottom equation by (-1) and then add the two equations). Then solve the resulting equation 2y = 8 to get y = 4.
<u>Step 2</u>: Substitute y = 4 into the top equation to find the corresponding x-value and get x = 3. The intersection point is (3, 4).
<u>Step 3</u>: Graph the four linear inequalities. First sketch the boundary lines 2x + 3y = 18 and 2x + y = 10. Then shade the correct half-planes. Label the boundary lines and the point of intersection found in Steps 1 and 2.
<u>Step 4</u>: Locate the vertices of the region determined by the overlapping shaded regions found in the

completed graph of Step 3.
Step 5: Evaluate the Objective Quantity for each of the three vertices found in Step 3 by substituting each of the x and y values into the expression $4x + 5y$. Then simplify.
Step 6: Choose the smallest number in the list of values from Step 5 to find the minimum value for the Objective Quantity function.

The minimum value is 36.

Note that the problem could have asked to determine the maximum value for the Objective Quantity. In the problem of Example 1, no maximum value could be found because the overlapping shaded region found in Step 3 is not a bounded region, that is, the overlapping region is not fully contained in a closed region such as the figure below. Thus there is no maximum value that can be found for $C = 4x + 5y$. If the region had been bounded, then each of the vertices of the bounded region would be substituted in the Objective Quantity, as shown by Step 5 above, to determine the maximum value. The largest number found would then give the maximum value.



Irrational Numbers: Pi
An irrational number is a number that cannot be written as a fraction. The decimal equivalents of irrational numbers do not terminate (end) and never repeat. For example, 0.10110111011110...and $\pi \sim$ 3.14159265... are two decimals that never repeat and never end.

Determining the Circumference of a Circle:

Circumference is the measurement of the distance around a circle. The circumference of a circle is found by using the following formula:

$$C = 2\pi r$$

In this formula, r represents the radius of the circle. The radius of a circle is the length of the segment from the center of a circle to any point on the circle. The radius is one-half the length of the diameter (the distance across a circle) of a circle. Pi is the ratio of the circumference of any circle to its diameter and is approximately equal to 3.14159. Pi is represented by the symbol π.

There is a key on the calculator that represents π. It may be necessary to press SHIFT, 2nd, or INV before the π key. If a calculator does not have a π key, it is acceptable to approximate π with 3.14.

Example 1: Find the circumference of a circle with a diameter of 18 inches.

(1) $r = 18 \div 2$

(2) $C = 2\pi \cdot 9$

(3) $C = 18\pi$

Step 1: Since the radius of a circle is one-half the diameter, divide 18 by 2 to get the value of the radius(9).
Step 2: Substitute 9 in place of r in the formula for the circumference of a circle.
Step 3: Multiply 9 by 2 (all three terms are multiplied and it does not matter which order numbers are multiplied).

The circumference is 18pi inches.

Example 2: Find the diameter of a circle whose circumference is 87 feet.

(1)	(2)	(3)	(4)
$87 = 2\pi r$	$\frac{87}{2\pi} = \frac{2\pi r}{2\pi}$ $\frac{87}{2\pi} = r$	$\frac{2 \cdot 87}{2\pi} = r \cdot 2$	$\frac{87}{\pi} = d$

Step 1: Substitute 87 in place of C because 87 is the circumference.
Step 2: To isolate the r on one side of the equation, divide each side of the equation by 2π Step 3: Since the radius is half the length of the diameter, multiply the radius by 2 to get the length of the diameter. Remember, what is done to one side of an equation must be done to the other side in order for the equation to balance, so multiply the left side of the equation by 2 also. The 2 in the numerator cancels with the 2 in the denominator on the left side of the equation.
Step 4: The diameter of the circle equals 87/pi feet.

Finding the Area of a Circle:

The area of a figure is the measure, in square units, of the interior region of a two-dimensional figure. The area of a circle is found by using the following formula:

$$A = \pi r^2$$

In this formula, r represents the radius of the circle.

Example 3: Find the area of the circle whose radius is 10.2 inches.

(1) (2)

$A = \pi(10.2)^2 \qquad A = 104.04\pi$

Step 1: Substitute 10.2 in place of r because the radius of the circle is 10.2.
Step 2: Square 10.2 (10.2 x 10.2) and multiply the answer by π.

The area of the circle whose radius is 10.2 is 104.04pi square inches.
Example 4: Find the area of the circle whose circumference is 35 inches.

(1) $C = 2\pi r$, $35 = 2\pi r$

(2) $\frac{35}{2\pi} = \frac{2\pi r}{2\pi}$

(3) $r = \frac{17.5}{\pi}$

(4) $A = \pi r^2$, $A = \pi\left(\frac{17.5}{\pi}\right)^2$

(5) $A = \pi \cdot \frac{(17.5)^2}{\pi^2}$

(6) $A = \frac{\pi}{1} \cdot \frac{306.25}{\pi^2}$

(7) $A = \frac{306.25}{\pi}$

Step 1: Before the area of the circle can be determined, the radius of the circle must be calculated. Since the circumference is known, the formula for the circumference of a circle can be used to determine the radius of the circle. Substitute 35 in place of C in the circumference formula.
Step 2: To isolate the r on one side of the equal sign, divide each side of the equation by 2π. Step 3: Divide 35 by 2 and the radius is $\frac{17.5}{\pi}$. Step 4: Now that the radius of the circle is known, it can be substituted in place of r in the formula for the area of a circle
Step 5: $\left(\frac{17.5}{\pi}\right)^2$ can be rewritten as $\frac{(17.5)^2}{(\pi)^2}$. Step 6: Square 17.5 (17.5 x 17.5) to get 306.25. Rewrite π as a fraction $\left(\frac{\pi}{1}\right)$. Step 7: The pi in the numerator and one pi in the denominator divide out and leave one pi in the denominator.

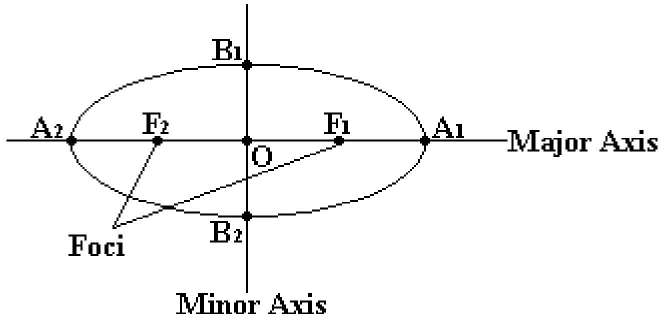
The area of the circle, whose circumference is 35 inches, is 306.25/pi square inches.

Finding the Area of an Ellipse:

An ellipse is a closed two-dimensional plane figure that is oval in shape. Every ellipse has two axes. The two axes lie on the symmetry lines and intersect at the center O of the ellipse. One of the axes is the major axis. The major axis contains the foci, has two vertices of the ellipse as its endpoints, and is always longer.

The length of the major axis is the distance from A_1 to A_2, where A_1 and A_2 are vertices of the ellipse.
The other axis of an ellipse is called the minor axis. The minor axis does not contain the foci and has two co-vertices of the ellipse as its endpoints.

The length of the minor axis is the distance from B_1 to B_2, where B_1 and B_2 are vertices of the ellipse.



The formula for an ellipse with axes of lengths 2a and 2b follows.

$$A = \pi ab$$

The length of the major axis is 2a and the length of the minor axis is 2b.

Example 5: Find the area of an ellipse whose minor axis is 23 feet and major axis is 35 feet.

(1) $2a = 35 \quad 2b = 23$

(2) $\frac{2a}{2} = \frac{35}{2} \quad \frac{2b}{2} = \frac{23}{2}$

(3) $a = 17.5 \quad b = 11.5$

(4) $A = \pi \cdot 17.5 \cdot 11.5$

(5) $A = 201.25\pi$

Step 1: The lengths of the axes can be used to determine a and b, so the formula for the area of an ellipse can be used. Since the major axis is 35 feet long, 2a = 35. The minor axis is 23 feet long, so 2b = 23.
Step 2: Divide each side of each equation by 2 to isolate the variable in each equation.
Step 3: Divide 35 by 2 to determine that a = 17.5. Divide 23 by 2 to determine that b = 11.5.
Step 4: Substitute the values of a and b into the formula for the area of an ellipse.
Step 5: Multiply 17.5 by 11.5 (numbers can be multiplied in any order) and multiply that product by π. The area of the ellipse is A = 201.25pi square feet.

Example 6: Find the minor axes of an ellipse whose major axis is 25 inches and whose area is 55 square inches.

(1) $\frac{2a}{2} = \frac{25}{2}$, $a = 12.5$

(2) $55 = \pi \cdot 12.5 \cdot b$

(3) $\frac{55}{\pi \cdot 12.5} = \frac{\pi \cdot 12.5 \cdot b}{\pi \cdot 12.5}$

(4) $\frac{4.4}{\pi} = b$

(5) $2 \cdot \left(\frac{4.4}{\pi}\right) = b \cdot 2$

(6) $2b = \frac{8.8}{\pi}$

Step 1: Since the area of the ellipse and the length of the major axis are known, the formula for the area of an ellipse can be used to determine the length of the minor axis. The length of the major axis (25) can be used to find the value of a, so 2a = 25. Divide each side of the equation by 2 to isolate the variable. 25 ÷ 2 = 12.5, so a = 12.5.
Step 2: Substitute the value of the area (55) and the value of a (12.5) into the formula for the area of an ellipse. We can use this new equation to solve for b (which is 1/2 the length of the minor axis).
Step 3: To isolate b on one side of the equal sign, divide each side of the equation by 12.5π. Step 4:

Divide 55 by 12.5 to determine that b = $\frac{4.4}{\pi}$. Step 5: Since the length of the minor axis is 2b and we know the value of b, we need to multiply each side of the equation by 2 to determine the length of the minor axis.

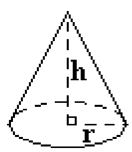
Step 6: Multiply b by 2 to get 2b. Multiply $\frac{4.4}{\pi}$ by 2 to get $\frac{8.8}{\pi}$.
The length of the minor axis of the ellipse is 8.8/pi inches.

Finding the Volume of a Cone:

A cone is a 3-dimensional figure with one curved surface, one flat surface (usually circular), one curved edge, and one vertex. The volume is the number of cubic units it takes to fill a figure. The formula for the volume of a cone is:

$$V = \frac{1}{3}\pi r^2 h$$

In this formula, r represents the radius of the circle (base) and h represents the height of the cone. (See the diagram of a cone below.)



Example 7: Find the volume of a cone that has a radius of the base equal to 8 inches and height equal to 12 inches.

(1) $\frac{1}{3}\pi\left(8^2\right)(12)$

(2) $\frac{1}{3}\pi(64)(12)$

(3) 256π

Step 1: Since the radius r and the height h are known, substitute the values into the formula for the volume of a cone.
Step 2: Following the order of operations, square the 8 first (8 x 8 = 64).
Step 3: Multiply 64 and 12, then multiply that product by 1/3. (64 x 12 = 768; 768 x 1/3 = 256)

The volume of the cone is V = 256pi cubic inches.

Example 8: Find the height of a cone that has a radius of the base equal to 10 inches and volume equal to 820 cubic inches.

(1)

$$820 = \frac{1}{3}\pi\left(10^2\right)h$$

(2)

$$820 = \frac{1}{3}\pi\left(100\right)h$$

(3)

$$\frac{820}{100\pi} = \frac{\frac{1}{3}\pi\left(100\right)h}{100\pi}$$

(4)

$$\frac{3}{1}\cdot\frac{8.2}{\pi} = \frac{3}{1}\cdot\frac{1}{3}h$$

(5)

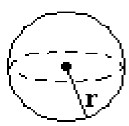
$$\frac{24.6}{\pi} = h$$

Step 1: Since the volume of the cone and the radius of the base are known, the formula for the volume of a cone can be used to determine the height of the cone. Substitute the value of the volume and the value of the radius into the formula for the volume of a cone.
Step 2: Following the order of operations, square the 10 (10 x 10 = 100).
Step 3: To begin to isolate the h on one side of the equal sign, divide each side of the equation by 100π. Divide 820 by 100 to get 8.2.
Step 4: Multiply each side of the equation by the inverse of 1/3, which is 3/1, to finish isolating the h on one side of the equation. The 3 in the numerator divides out with the 3 in the denominator.
Step 5: Multiply 8.2 by 3 get 24.6.

The height of the cone is 24.6/pi inches.

Finding the Volume of a Sphere:

A sphere is a 3-dimensional figure made up of all points in space that are equally distant from a given point called the center. The best example of a sphere is a ball.



The formula for the volume of a sphere is:

$$V = \frac{4}{3}\pi r^3$$

In this formula, r represents the radius of the sphere.

Example 9: Find the volume of a sphere whose radius is 11 inches.

(1) $V = \frac{4}{3}\pi\left(11\right)^3$

(2) $V = \frac{4}{3}\pi\left(1331\right)$

(3) $V = 1774.67\pi$

Step 1: Substitute the value of the radius (11) into the formula for the volume of a sphere.
Step 2: Following the order of operations, calculate $11^3 = 11\times 11\times 11 = 1,331$. Step 3: Multiply 1331 by 4/3. To do this, first multiply 1331 by 4, then divide the product by 3 (1331 x 4 = 5324; 5324 ÷ 3 ~ 1774.67).

The volume of the sphere is V = 1.774.67pi cubic inches.

Example 10: Find the radius of the sphere whose volume is 1,432 cubic feet.

(1)
$$1432 = \frac{4}{3}\pi r^3$$

(2)
$$\frac{3}{4}\cdot 1432 = \cancel{\frac{3}{4}}\cdot\cancel{\frac{4}{3}}\ \pi r^3$$

(3)
$$\frac{1074}{\pi} = \frac{\pi r^3}{\pi}$$

(4)
$$\sqrt[3]{\frac{1074}{\pi}} = \sqrt[3]{r^3}$$

(5)
$$r = \sqrt[3]{\frac{1074}{\pi}}$$

Step 1: Since the volume of the sphere is known (1432), the formula for the volume of a sphere can be used to find the radius of the sphere. Substitute the value of the volume of the sphere into the formula for the volume of a sphere.
Step 2: Multiply both sides of the equation by the reciprocal of 4/3, which is 3/4, to begin isolating the r. To multiply 1432 by 3/4, multiply 1432 by 3 then divide that product by 4 (1432 x 3 = 4296; 4296 ÷ 4 = 1074).
Step 3: Divide both sides of the equation by pi to isolate the r^3. Step 4: Take the cube root of each side of the equation. This will isolate the r on one side of the equal sign.
Step 5: Once the r has been isolated, the equation can be written beginning with the r. It is acceptable to leave the radical sign in the answer.

The radius of the sphere is $r = \sqrt[3]{\frac{1074}{\pi}}$ feet.